Revised Optical Properties of Turbid Media on a Base of Generally Improved Two-Flux Kubelka-Munk Approach

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Abstract - On the base of the generally improved Kubelka-Munk approach what now allows to obtain exact solutions for backscattered and transmitted fluxes on external boundaries of a turbid medium in a one-dimensional theoretical problem some main definitions of optical properties of turbid media have been revised. It is proved by numerical calculations that the more light absorption is presented in the scattering medium the more incorrect values can be obtained for transport optical properties using the classical approach. The article proposes new strict definitions of the transport scattering and absorption coefficients as well and shows that it is more logically to distinguish along with them still factors like a transport attenuation coefficient (*AC*) and a transport backscattering coefficient (*BC*). The ratio *BC/AC* defines exactly the albedo of the turbid medium.

1. INTRODUCTION

Kubelka-Munk (KM) two-flux transport one-dimensional (1D) model is the most widely used Radiation Transport Theory (RTT) approach in a modern optics of turbid media because of its simplicity and existence of a clear and analytical solution of initial differential equation in it [1, 2]. Moreover, the KM approach is the best and the simplest approximation of the general Radiative Transport Equation (RTE) in the case of 1D theoretical problems [3]. But it is well-known from the literature that the KM model doesn’t allow to obtain an exact solution, especially for highly-absorbing and weakly-scattering media [3, 4]. In most of publications it is assumed that light must be diffuse on a surface as well as within the medium for a correct application of the KM equations. As the main consequence of quite simultaneously and probably independently appeared publications [5, 6] in the case of diffuse light distribution there is a totally accepted opinion that the ratios between transport optical properties of KM approach (*K* and *S*) and corresponding transport optical properties of general RTE (μ_a and μ_s) can be written, for example [6], as :

$$K \approx 2\mu_a \quad ; \qquad S = \frac{3}{4}\mu_s - \mu_a \quad . \tag{1}$$

Herein, if the second equation (1) needs

$$\mu_a < \frac{3}{4}\mu_s$$

what is usually explained like a necessary consequence of highly-scattering and diffuse scattering conditions, the first equation of (1) in the case of small scattering looks more dramatically for doesn’t contains any dependences on μ_s. In a case of $\mu_s \to 0$ both KM and RTT equations became identical and have an exponential attenuation of light intensities as a solution of equations. These analytical-precise two solutions cannot differ in two times, so, ether $K=\mu_a$ all time and the first equation of (1) is wrong or *K* is unknown function of μ_s when if $\mu_s=0$ then $K=\mu_a$ but if $\mu_s \neq 0$ then *K* aspires to (1) under $\mu_s >> \mu_a$.

All these and some other deficiencies of a classic KM approach were a source of inexhaustible attempts to improve a physical interpretation and an accuracy of the classic KM approach all time during last 50 years [7-9]. Recently it was shown [10-12] that the main problem of the KM as well as of the general RTE approaches consists in a wrong phenomenological assumption of the existence of two independent optical transport properties of turbid media - absorption and scattering. In the general case of a turbid medium, where the absorption of light is presented, the first coefficient in the right side of both RTE and KM equations cannot be separated into the two independent transport coefficients - absorption and scattering (*K* and *S* in the KM notations; μ_a and μ_s in the RTE notations) - and must be considered as one, united attenuation coefficient "β_1". The absorption transport property K (or μ_a in RTE notations) is included into β_1 as well as into the second coefficient of initial differential equations "β_2", but not additively. Without absorption $K=0$ and $\beta_1=\beta_2=S$; without scattering $S=0$, $\beta_2=0$ and $\beta_1=K$, like it must be in the classical theory. But if both an absorption and scattering phenomena are presented together, then the classical phenomenological assumption $\beta_1=K+S=K+\beta_2$ is wrong. Only if the absorption existed in a medium is small, much less than the scattering, then the classical assumption can take place.

Thus, the generally improved two-flux KM approach shows to us that there are in the RTT a number of incorrect-understandable definitions of transport optical properties of turbid media. For example, - the definition of the scattering properties of the medium or the definition of *albedo* (W_0) of that. The latter is frequently understood as the ratio $W_0=\beta_2/\beta_1=S/(K+S)$ [3], but as it has been shown [10, 11] in the general case of 1D task $\beta_2/\beta_1\neq S/(K+S)$. So in this work we have studied more detailed a difference between classic and improved values for transport coefficients and terms mentioned above.

2. CLASIC AND GENERAL IMPROVED TWO-FLUX KUBELKA-MUNK APPROACH

The original (classic) two-flux KM model is a well-known approach with two fluxes $F_+(x)$ and $F_-(x)$ of light traveling in a turbid medium in forward and backward directions. The optical properties of the medium are described by transport absorption (K) and scattering (S) coefficients. Initial KM differential equations describing the radiant energy balance on dx of the medium are:

$$\begin{cases} dF_+(x)/dx = -(K+S)F_+(x) + SF_-(x) \\ dF_-(x)/dx = (K+S)F_-(x) - SF_+(x) \end{cases} . \qquad (2)$$

System (1) has an exact and analytic-mathematical solution:

$$F_+(x) = C_1 e^{-\alpha x} + C_2 e^{\alpha x} \quad ; \qquad F_-(x) = C_1 A_- e^{-\alpha x} + C_2 A_+ e^{\alpha x} \quad , \qquad (3)$$

where: C_1 and C_2 are the constants of integrating; $\alpha = \sqrt{K(K+2S)}$; $A_+ = (K+2S+\alpha)/(K+2S-\alpha)$; $A_- = 1/A_+$. Usually for various practical applications it is interesting to find a backscattered flux $F_{bs}= F_-(0)$ or a transmitted one $F_\tau= F_+(H_0)$, where H_0 is the thickness of the medium.

The generally improved two-flux KM approach differs from the system (2) in its coefficients of equations:

$$\begin{cases} dF_+(x)/dx = -\beta_1 F_+(x) + \beta_2 F_-(x) \\ dF_-(x)/dx = \beta_1 F_-(x) - \beta_2 F_+(x) \end{cases} . \qquad (4)$$

Moreover, as it has been shown in ref. [10, 11] in the general case of the problem for the system (4) these coefficients have much more complicated forms, counterbalancing (2):

$$\beta_1 = \omega \cdot \frac{\mu_a - \mu_\rho \ln(1-R) + \mu_\rho \ln(1-\omega+\sqrt{\omega^2 - R^2 e^{-2\mu_a/\mu_\rho}})}{\sqrt{\omega^2 - R^2 e^{-2\mu_a/\mu_\rho}}} \; ; \qquad (5)$$

$$\beta_2 = R \cdot e^{-\mu_a/\mu_\rho} \cdot \frac{\mu_a - \mu_\rho \ln(1-R) + \mu_\rho \ln(1-\omega+\sqrt{\omega^2 - R^2 e^{-2\mu_a/\mu_\rho}})}{\sqrt{\omega^2 - R^2 e^{-2\mu_a/\mu_\rho}}} \; ; \qquad (6)$$

$$\omega = \frac{1-(1-2R)\cdot e^{-2\mu_a/\mu_\rho}}{2} \; ; \qquad (7)$$

and in the solution (3) $\alpha = \sqrt{\beta_1^2 - \beta_2^2}$, where μ_ρ is a transport density of optical heterogeneities in the medium per length "dx" and R is a coefficient of reflection on boundaries of the heterogeneities (in a 1D problem there is a single physical phenomenon only – the reflection - to change the direction of light propagating in the medium). What is important else: the results (5)-(7) are valid both for highly and weakly-scattering media.

As anyone can see now in a common case we can't write directly $\beta_1 = k\mu_a + \beta_2$ where k is any constant, because as it follows from (5) and (6):

$$\beta_1 = \frac{\omega \cdot e^{\mu_a/\mu_\rho}}{R} \cdot \beta_2 \quad . \tag{8}$$

Nevertheless, there are in the theory a number of more simplified cases which could be considered as limits of equations (5)-(7) when the transport optical properties could be written more closely to classical definitions. For example, in the case of $K=0$ and multiple scattering it was shown [10] that

$$\beta_1 = \beta_2 = \frac{R\mu_\rho}{1-R}, \tag{9}$$

what could be accepted like a transport scattering coefficient "S_m" in a perfect multiple scattering 1D problem. But in the case of a small diffusing when an approximation of the single scattering is applied [12]:

$$\begin{aligned} \beta_1 &= \mu_a - \mu_\rho \cdot \ln(1-R) \\ \beta_2 &= \frac{R \cdot e^{-\mu_a/\mu_\rho}}{1 - R \cdot e^{-2\mu_a/\mu_\rho}} \cdot (2\mu_a - \mu_\rho \ln(1-R)) \end{aligned} \quad , \tag{10}$$

where the division of the coefficients β_1 and β_2 into two classical transport optical properties K and S is not so obviously. And, definitely, equations (1) contradict to the exact and new general results (5)-(8).

3. REDEFINITION OF OPTICAL PROPERTIES OF TURBID MEDIA

To have a clear idea about transport optical properties of turbid media in a light of our results (5)-(8) anyone now has to decide what to consider as the absorption transport coefficient of the medium and the scattering one? There are a number of opportunities, for instance [10]:

1. $$K = \mu_a \quad \text{and} \quad S_m = \mu_s = \frac{R\mu_\rho}{1-R}. \tag{11}$$

These definitions are more natural in both cases of perfect absorption and perfect scattering because of the existence of corresponding limits of (5) and (6). For example, in the case of $\mu_\rho \to 0$ (or $R \to 0$) $\Rightarrow$ $\beta_2 \to 0$ and $\beta_1 \to \mu_a$ (the perfect absorption), as well as in the case of $\mu_a \to 0$ (perfect scattering) there is the solution (9) [12].

2. $$K = \mu_a \quad \text{and} \quad S = \mu_S = -\mu_\rho \cdot \ln(1-R) \tag{12}$$

In this case under the single scattering approximation $\beta_1 = K+S$ what follows from first equation (10), but $\beta_2 \neq S$.

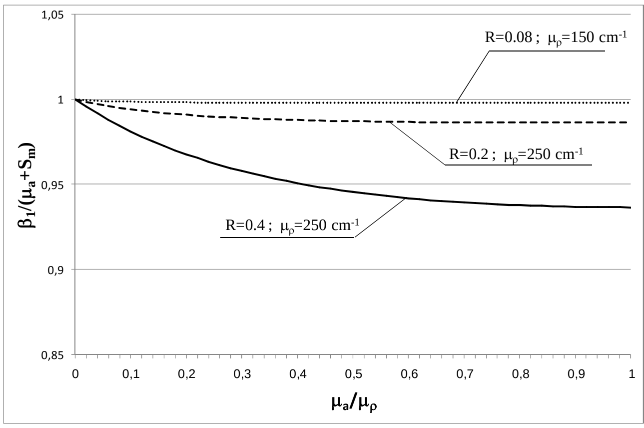
3. $K = \mu_a$; $S=\beta_2$ but $\beta_1 \neq K+S$, etc.

It has to be special noted that in the system (4) there is one more optical parameter $J=\beta_1/\beta_2=const\geq 1$ which is important and could be numerical determined from eq. (8). In ref. [13] it had been defined as *one of the photometrical invariants*. However, in the light of our problem if we suppose the albedo $W_0=\beta_2/\beta_1$, then $W_0=1/J$ and the albedo is the simplest inverse quantity to this *Gershun-Gurevich Invariant*.

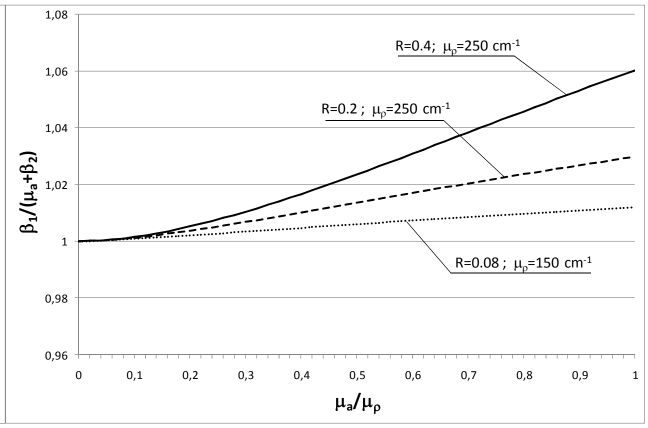
In our opinion, more logically is to have definitions of transport optical properties in the forms (11) or (12). Additional optical coefficients – coefficients of initial transport differential equations (4) – could be called like the transport attenuation coefficient (β_1) and the transport coefficient of a backscattering (β_2). At last the albedo we'd suggest to name $W_0=\beta_2/\beta_1=1/J$, refusing classical $\beta_1=K+S$ as well as $S=\beta_2$ and $W_0=S/(K+S)$.

4. NUMERICAL EXAMPLES AND RATIOS

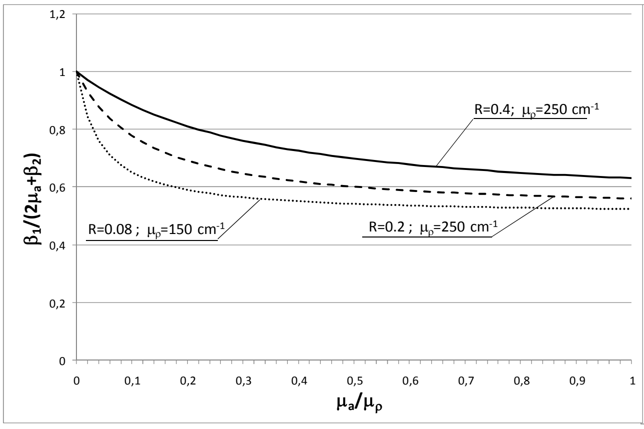
To understand better some differences in numerical values of various definitions of transport optical properties of turbid media we have calculated their ratios for different sets of parameters R, μ_a and μ_ρ. The results are shown in Fig. 1 and 2. Evidently, the presence of absorption in the medium ($\mu_a \neq 0$) leads to difference between classical results and exact new ones. The more μ_a exists in the medium the more errors we can see in the classical calculations and definitions.



a)



b)



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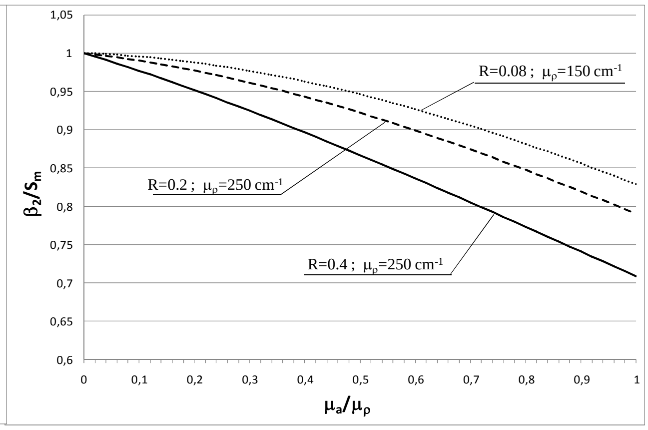

d)

Figure 1. Differences in transport optical properties under different definitions of them.

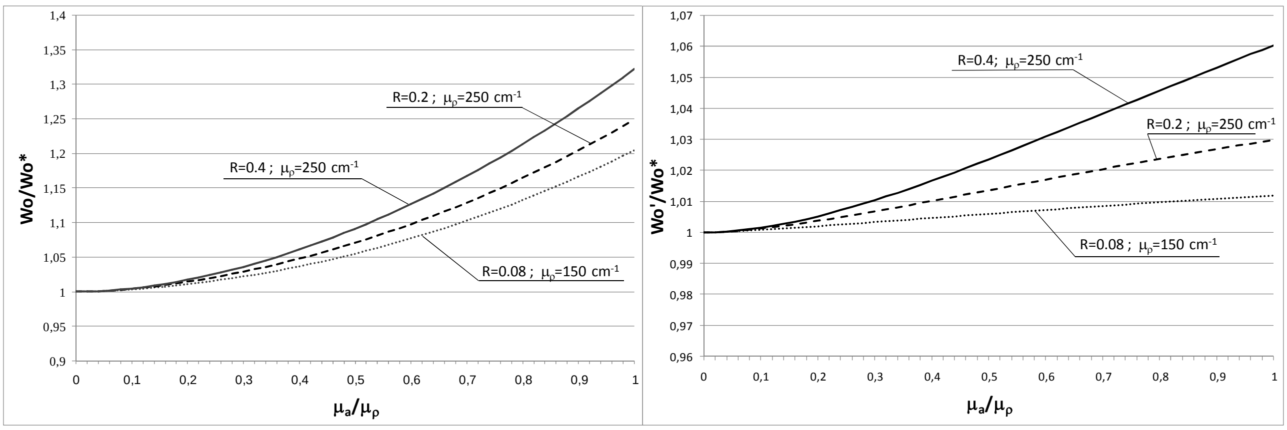

Figure 2. Differences in numerical values of different meanings of albedo.
$W_0=S_m/(\mu_a+S_m)$; $W_0{}^*=\beta_2/\beta_1$; $W_0{}' = \beta_2/(\mu_a+\beta_2)$.

5. DISCUSSION AND CONCLUSION

In our opinion, we have obtained quite interesting results, especially equations (5)-(8). They show that in the classical RTT there are a number of problems when we can't separate easily the absorption and scattering transport coefficients. Such separation and the theoretical independence of *K* on *S* in the classical RTT is a simple consequence of the accepted radiant-energy balance formalism in the elementary volume of the scattering medium. But in a number of cases, for our 1D model in instance, both absorption and scattering processes are not independent. Moreover, our results come into operation the so-called in photometry (and today quite forgotten) *Gershun-Gurevich Invariant* [10, 13]:

$$\frac{1+(F_{bs}/F_0)^2-(F_\tau/F_0)^2}{2(F_{bs}/F_0)}=\beta_1/\beta_2=J=const \quad ,$$

where F_0 is an external illuminating flux. All of these reasons insist on a redefinition of some transport optical properties of turbid media in the RTT. If to have a look at figures 1 and 2 then it becomes evidently that classical formulas are not correct in the case of nonzero absorption. The more μ_a exists in the medium the more errors we can obtain in classical calculations. So, to have more physically understandable results it is necessary to redefine some transport optical properties in the classical RTT. Now we can propose and try to prove our vision of the problem. According to that we have suggested some new definitions for transport optical properties of turbid media. In our opinion, it is more logically to have definitions of transport optical properties in the forms (11) or (12). Additional optical coefficients – coefficients of initial transport differential equations (4) – could be called as a transport attenuation coefficient (β_1) and a transport coefficient of backscattering (β_2). At last the albedo we'd like to propose to name $W_0=\beta_2/\beta_1=1/J$, refusing classical $\beta_1=K+S$ as well as $S=\beta_2$ and $W_0=S/(K+S)$.

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