On one simple backscattering task of the general light scattering theory

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ABSTRACT

It is well known that there are some difficulties with the analytical solution of the main equations in the general light transport and scattering theory, which is widely used today in the biomedical optics and optical noninvasive medical diagnostics, in particular, if the optical turbid media are taking into account. As we have reported in some our previous publications we assume that marked problems in biomedical optics follow from the not quite correct formulation of main equations of the classical transport theory in application to biomedical noninvasive diagnostic tasks. To study more detailed this problem we had taken into consideration one simple and 1D theoretical modeling task for which we have tried to obtain the simple analytical solution for the backscattered flux. This report highlights the way to obtain expected result and differences between classical and our modified approaches.

Keywords: light, scattering, backscattering, transport theory, Kubelka–Munk approach, single scattering approach.

1. INTRODUCTION

The latest developments in different branches of medicine exist on application of physical and mathematical methods, computers, electronics, etc. to a medical practice, which lead to appearance of high and precision technology in medicine. One of such way is the application of laser and other optical techniques to medical noninvasive (in situ, in vivo) diagnostics. At a present time there are a number of well-known optical and laser methods of diagnostics which are efficient in medicine: biophotometry, photoplethysmography, fluorescent diagnostics, oximetry, elastic scattering spectroscopy, etc.[1-3]. The key point in a realization of all these methods is the mathematical calculation algorithms basing on methods of the transport and a light scattering theory for optically turbid media, which allow to receive using the measured data the important information for the doctor about a biochemical and morphological structure of patient’s soft tissues[4]. Accordingly, accuracy and veracity of the used calculation algorithms and computational physical and mathematical models play a large role in implementation of diagnostic process and determine an accuracy and veracity of all spent diagnostics.

Between that, it is well known, that the solution of the basic equations in the classical transport theory is integrated to a number of difficulties. Generally, for these equations it is not retrieved yet a precise analytical solution, that results in necessity of the use of different approaches lowering general veracity of medical diagnostic result, received on their basis.

As we repeatedly informed in our publications[5,6], in our opinion the existing difficulties to obtain precise analytical solutions in the transport theory are connected with a general incorrectness in the original phenomenological formulation of the base concepts and equations of this theory. Especially it becomes noticeable in the application to the optical noninvasive medical diagnostic problems, when the so-called backscattered radiation, which comes out the frontal surface of the medium, is registered and processed. By this paper we’d like to show, that in the transport theory the alternate approach to classic methods for the formulation of the basic equations to describe a coming out backscattering radiation can exist. It can allow, herein after, to receive more simple and analytically precise solutions for simulations tasks of diagnostics.

2. GENERAL THEORY

We shall esteem a one-dimensional (1D) photometric problem of a radiation transfer in light-diffusing medium in the approach of single scattering. The selection of such simple model is not accidental and is determined by a series of essential advantages. At first, the 1D model allows to distract on time from padding difficulties, bound with definition of phase scattering functions in multidimensional problems of the transport theory, and to be massed, mainly, on the phenomenological fundament of the theory. Secondly, the 1D models of the transport theory being known more as a fluxes model by Kubelka-Munk (KM)[7], nevertheless, has the precise and analytical solutions for base tasks, and it makes a very convenient comparison with it another results based on any other new methods and approaches. Other cause, that in the literature it has taken roots the generally accepted opinion about disharmony of the results of calculations on the basis of KM models with results of the similar problem's solution on the basis of a general transport equation[7]. But in our case, contrary, it is the one more padding argument for the benefit of a selection just only 1D models, as, studying them, it is possible to attempt to find out more steep problems inside the theory, connected with the initial phenomenology of a problem. Running forward, it is possible to say, that exactly with the use of the 1D model of single scattering approach it most visually can be shown, that the KM model by means of properly selection of the transport coefficients for a medium actually give precise and correct analytical solution of the problem as well.

The general formulation of our 1D scattering task in the application to the problems of optical diagnostics looks like (Fig.1). Light-diffusing semi-translucent medium is illuminated by an external light flux F_0. The radiation penetrates medium along axis "x" and while undergoing inside medium is absorbed and scattered by heterogeneities of an inner structure. It is necessary to calculate a flow of backscattered radiation F_{BS} as a function of real optical properties of the medium.

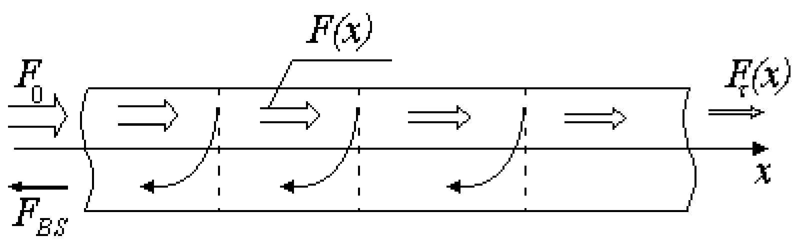

Fig. 1: The general formulation of 1D scattering task

Before to start the solution of the given problem we shall make some preliminary remarks concerning the model description of absorption and scattering processes in this 1D medium and of bounded with it ways to describe transport optical properties of the medium. Let's remark at once that the radiation scattering in a 1D case from the point of view of physics phenomena can descend only because there is a Fresnel's reflection of radiation from borders of non-uniformity of the medium. The concept of diffraction of radiation in a 1D case is dispossessed of any sense that essentially simplifies an initial formulation of the task and allows anyone to describe by a rather simple mode a radiation scattering into the medium. The 1D scattering medium can be in this case simply simulated by the fixed section of a straight line, in definite points (coordinates) of which one arranges borders of heterogeneities exist, from which the radiation, passing in medium, is partial mirrored and starts to be passed backwards (Fig.2). Phenomenology of the transport theory leave outs the wave properties of light as well, so the straight flux F and back flux F_{BS} can be considered not interfering between each other and, so, distributed separately from each other.

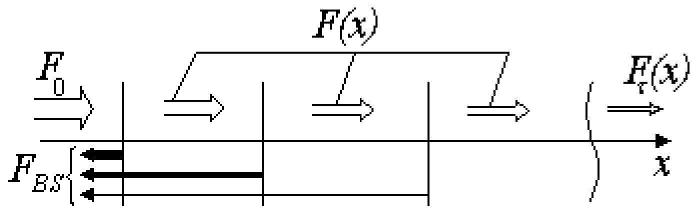

Fig. 2: Model of the backscattering flux forming.

Let, further, all conditional heterogeneities of our medium have got an identical optical properties, i.e. the radiation is mirrored on a border of any of heterogeneities with the same reflection coefficient R and passes through it with a factor *(1-R)*. Let our medium has finite depth H_0 and total amount of heterogeneities on this depth N (Fig.3).

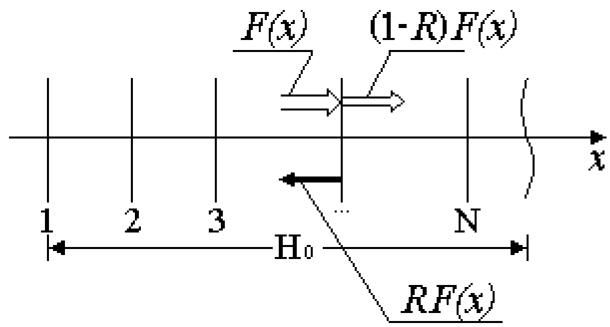

Fig. 3: A 1D medium with depth of H_0 and N identical scatters heterogeneities.

If the perfect scattering medium is esteemed, i.e. the medium without absorption, and processes of the re-reflection of radiation between any two heterogeneities inside the medium are not considered, then the fundamental solution of the transport theory and the KM method for the transmitting such medium radiation is the elementary equation of a kind:

$$F(H_0) = F_0 e^{-SH_0}, \tag{1}$$

where F_0 - external radiation flux, dropping on medium, $F(H_0)$ - radiation flux, which transmits a layer of medium of depth H_0, S - per unit length scattering coefficient for the medium (transport scattering coefficient)

This equation is similar of the Bouguer's equation for a perfect scattering medium.

Ours model submission about an inner pattern and descending optical processes inside such medium allows to write a solution for the transmitted flux in a little bit diverse kind:

$$F(H_0) = F_0 (1 - R)^N, \tag{2}$$

where R is the reflection coefficient.

Whether these two expressions (1) and (2) are equivalent? Let's remark, that if to enter under our consideration a mean density of heterogeneities inside the medium

$$\mu_\rho = \frac{N}{H_0}. \tag{3}$$

And to consider a certain medium's layer with the depth H_0, then by equating (2) and (1) we shall express *S* as:

$$S = -\mu_\rho \ln(1-R). \tag{4}$$

Under any other choose of *S* the solution (1) and (2) will be differ from each other.

Similarly we shall do for the description of the absorption process into the medium. If our medium consists on a number of linear absorbers and each absorber absorbs a part "*a*" $(0 \le a < 1)$ of the incident flux on interval Δx =H_0/N, then we can write for the KM approach:

$$F(H_0) = F_0 e^{-KH_0}. \tag{5}$$

And for our approach:

$$F(H_0) = F_0(1-a)^N, \tag{6}$$

where K – transport absorption coefficient. By equating (5) and (6) we can get:

$$K = -\ln(1-a) \cdot \mu_\rho. \tag{7}$$

Now we can start the main solution of our task. As opportunity to the classic transport theory approach and the method KM we shall not search for expressions for direct and inverse radiation fluxes inside the medium, but we attempt to formulate at once a differential equation to find out a backscattered flux F_{BS}. According to the classic definition of derivative:

$$\frac{dF}{dx} = \lim_{\Delta x \to 0} \frac{\Delta F}{\Delta x}. \tag{8}$$

So to obtain the dF_{BS}/dx it is necessary to determine increment of this flow with Δx of medium. As a first step we shall consider a perfect scattering medium without absorption. In single scattering approach it is supposed, that during radiative transfer in a positive direction of axis "*x*" light undergoes reflection on heterogeneities of the medium and as a result of it is attenuated. The reflected radiation, being diffused backwards, is not dispersed any more on heterogeneities of medium and, accordingly, is not attenuated. As mean density of heterogeneities inside medium we have determined in (3), any Δx contains $\Delta x \cdot N / H_0$ heterogeneities. Let us select mentally inside the medium an interval $\Delta x > H_0/N$. The increment ΔF_{BS} of a back flux with increment Δx will be equal to a flux leaving the Δx in a negative direction of axis "*x*", if on the Δx there is the coming flux $F(x)$. For our model medium without absorption a leaving flux and, therefore, ΔF_{BS} are:

$$\Delta F_{BS} = F(x) \cdot R \cdot \left[\sum_{i=1}^{\mu_\rho \Delta x} (1-R)^{i-1} \right]. \tag{9}$$

With the use of (2) and (3):

$$F(x) = F_0 (1-R)^{\mu_\rho x} . \tag{10}$$

Thus,

$$\Delta F_{BS} = F_0 (1-R)^{\mu_\rho x} \cdot R \cdot \left[\sum_{i=1}^{\mu_\rho \Delta x} (1-R)^{i-1} \right]. \tag{11}$$

Let's remark, that in brackets the members of the sum of a series represent the decreasing geometrical progression with a multiplicator *(1-R)<1*, so the finding of this sum does not introduce the special complexity:

$$\sum_{i=1}^{\mu_\rho \Delta x} (1-R)^{i-1} = \frac{1-(1-R)^{\mu_\rho \Delta x}}{R} . \tag{12}$$

Then with the use of (11) and (12) let's find now the $\Delta Fbs/\Delta x$:

$$\frac{\Delta F_{BS}}{\Delta x} = \frac{F_0 (1-R)^{\mu_\rho x} \cdot \left[1-(1-R)^{\mu_\rho \Delta x}\right]}{\Delta x} . \tag{13}$$

Under $\Delta x \to 0$ expression (13) represents indeterminacy of a kind 0/0 and the limit of (13) can be found by the rule of L'Hospital:

$$\lim_{\Delta x \to 0} \frac{\Delta F_{BS}}{\Delta x} = -\mu_\rho F_0 (1-R)^{\mu_\rho x} \cdot \ln(1-R) = \frac{dF_{BS}}{dx} . \tag{14}$$

So, the general solution of our problem is:

$$F_{BS} = \int_0^{H_0} (-\mu_\rho F_0 (1-R)^{\mu_\rho x} \cdot \ln(1-R)) dx = F_0 (1-(1-R)^{\mu_\rho H_0}) . \tag{15}$$

Or, with allowance of (4), one can write:

$$F_{BS} = F_0 (1 - e^{-SH_0}) \tag{16}$$

It is necessary to say, that such result for a perfect scattering medium was, generally speaking, rather anticipated, and is, naturally, a consequent of expressions (1-4) in the absence of absorption (all not transmitted by the medium radiation have to leave it backward). The same results also can be obtained with the use of standard KM approach if to take into consideration our expression (4). In this sense, apparently, result (16) is trivial. However this result, besides it is indispensable for us for a further analysis, already has one remarkable feature: at integrating (15) there are not any integrating constants and, accordingly, there is no necessity for the use of any padding prior boundary condition as against other known methods, for example - KM method, when the backward flux inside the medium is esteemed and for that the additional boundary condition on a right side of a medium is estimated.

Now we shall consider the same problem, but with the presence of absorption of radiation on each interval between heterogeneities inside the medium. Let's suspect for a simplicity of calculation that all intervals between heterogeneities are the same length and have identical absorptive properties. Using an offered technique with the use of obtained ratios (9-15) it is easy to receive the following solution for such complex task in a term of backscattered flux:

$$F_{BS} = \frac{RF_0}{1-Y}\left(1 - Y^{\mu_\rho H_0}\right), \tag{17}$$

where *a* – a part of radiation absorbed on any interval between heterogeneities and $Y = (1-R)(1-a)^2$.

It was supposed that the radiation is absorbed on intervals between heterogeneities both at its straight direction and back. If $a = 0$ these solutions completely coincide with (16). The reference approach of KM in our approximation of single scattering (taking into account dispersion of a straight flux in the medium and not taking one for a back flux) guesses solution of the system of the differential equations:

$$\begin{cases} \dfrac{dF_+(x)}{dx} = -(K+S)F_+(x) \\ \dfrac{F_-(x)}{dx} = KF_-(x) - SF_+(x) \end{cases}, \tag{18}$$

where K - transport absorption coefficient in the medium, S – transport scattering coefficient, $F_+(x)$ - radiation flux in the medium, spreading in a straight direction, $F_-(x)$ - radiation flux in the medium, spreading in a back direction.

System (18) has the following solution for the F_{BS}:

$$F_{BS} = \frac{SF_0}{2K+S} \cdot (1 - e^{-(2K+S)H_0}). \tag{19}$$

To have identical (17) and (19) the transport coefficients of the medium must be presented as follows:

$$\begin{aligned} S &= \frac{R}{1-Y}(-\mu_\rho) \cdot \ln Y \\ K &= \frac{1-Y-R}{2(1-Y)}(-\mu_\rho) \cdot \ln Y. \end{aligned} \tag{20}$$

Thus, it is possible to see, that the dependence of transport coefficients in the classical KM model on actual optical parameters of the medium ("R" and "a") under count of absorption is essentially complex. Both coefficients depend on both the absorbing and the scattering properties of the medium. Moreover, the dependences (20), satisfying to solutions for a backscattered flux, don’t obey to solution of the same task for a flux passing through the medium.

The solution of the system (18) for $F_+(H_0)$ as for $F_\tau(H_0)$ has kind:

$$F_{\tau}(H_0) = F_{+}(H_0) = F_0 e^{-(K+S)H_0}. \tag{21}$$

Our method gives solution as:

$$F_{\tau}(H_0) = F_0(1-R)^{\mu_\rho H_0}(1-a)^{\mu_\rho H_0}. \tag{22}$$

And for equality (21) and (22) it is necessary to save relations for transport coefficients of the medium in the forms (4) and (17), which, besides give unique and separate dependence of the coefficients on real absorption and scattering properties.

So the conflict in the definition of parameters of the medium for the KM model is observed. Our solution, being strict and precise for a 1D model, gives, on our point of view, more simple and understandable result.

3. DISCUSSION

Let's discuss the obtained results. In first, we shall remark, that the solutions (15) and (17) are strict and precise solutions of the task within the framework of the used approximation of a single scattering, because at their output it was not made of any padding simplifications and allowances. Moreover, in an offered method the differential equation for the backscattered flux is shaped at once.

In second, the comparison of outcomes of the offered method and of the classical KM approach displays that the transport coefficients of the medium in a KM method have generally rather composite dependence on real optical properties of the medium (20), that is, they are not real, but are “effective” and “average” parameters of the medium.

Recently we informed of obtained similar outcomes by reviewing a similar 1D problem without absorption, but with the multiple scattering into the medium[6]. There we have obtained a condition of a transport scattering factor S as follows:

$$S = \frac{R\mu_\rho}{1-R}. \tag{23}$$

The comparison (23) with (4) and (20) and their changes in depending on formulation of the task visually demonstrates to us, that the parameters S and K of the model of KM substantially are more the *parameters of the model*, rather than of real optical properties of the medium.

Besides that there is an evident and depressing contradiction between result (20) and equations (21) and (22). As our solutions (17) and (22) are mathematically strict and precise, it is necessary to assume, that in the initial formulation of KM model there is an initial incorrectness, which we also informed earlier about[6] and which results in appearance of the indicated inconsistency. This conflict is absent for the medium without absorption and occurs in a count of absorption. To avoid this situation it is possible to offer the initial system (18) formulation as:

$$\begin{cases} \dfrac{dF_{+}(x)}{dx} = -(K+S)F_{+}(x) \\ \dfrac{F_{-}(x)}{dx} = KF_{-}(x) - S'F_{+}(x) \end{cases}, \tag{24}$$

where S’ is the additional scattering transport coefficient, not equal to S.

The solution (24) has the similar to (19) form:

$$F_{BS} = F_{-}(0) = \frac{S' F_0}{2K + S}(1 - e^{-(2K+S)H_0}). \quad (25)$$

But equations for the coefficients of these two models with the use of (25) accept the simple and physically understandable form satisfying, in that number, the solution for the backscattering flux as well as for the transmitted:

$$\begin{aligned} K &= -\mu_\rho \ln(1-a) \\ S &= -\mu_\rho \ln(1-R) \cdot \\ S' &= \frac{R}{Y'}(2K+S) \end{aligned} \quad (26)$$

Where $Y' = 1-(1-R)\cdot(1-a)^2$

Thus, we see, that in the classical model of KM (the same we can say for the general classical transport theory as well) under the presence of absorption in the medium the full conversion of direct flux to the back flux is incorrectly postulated. Actually, because of absorption not all scattered direct flux is conversed to a radiation flux spread in the opposite direction. The part of it is absorbed in the medium already at a stage of conversion. The acceptance of such allowance reduces in complete identity of solutions in the precise method, offered by us, and in the KM model, however model of KM, thus, becomes complicated a little bit by introduction of the third parameter S'. Such position of things with the initial incorrectness of the equations of KM model, probably, was not remarked up to now in the classical transport theory. The taking into account our remarks can influence on processing of outcomes of experiments on definition of optical properties of biotissues, as the overwhelming majority of outcomes of experiments in the biomedical optics is processed today on the basis of a classical KM method.

4. CONCLUSION

In connection with existing problem to obtain the closed form, analytical solutions in the classical light transport theory for turbid media the search of the ways to develop the theory is rather actual. In this paper on example of a simple 1D problem in approximation of a single scattering the possibility of an alternate formulation of the initial differential equations of the theory to determine a backscattering radiation, coming out the medium from its frontal surface had been studied. It was shown, that the statement of the task is possible, when the initial differential equation is the single differential equation for a backscattered flux directly on a surface of the medium. Moreover, the comparison between the our results and results obtained on the basis of reference flux KM model, has revealed for KM a lot of important features and problems. It is shown too, that the transport parameters of the medium for KM model are actually "effective" and "average" parameters depending on a form of initial formulation of the task as well. Besides that in reference KM model the complete conversion of a direct flux in the flux spread in the opposite direction is incorrectly postulated. It reduces in composite dependence of transport coefficients of the model on real optical properties of the medium and does not allow to utilize the same coefficients at the same time in solutions for straight and back fluxes. For elimination of this lack of classical KM model a modification of it is offered on the basis of introduction of a padding scattering transport coefficient.

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