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New form of the transport equation for the case of 2D orthogonal scattering approximation in biooptics

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***Abstract* – Extension of the 2-flux Kubelka-Munk approach to a 2D radiative transfer problem was studied. New transport equation of the 4-th order for the case of orthogonal-scattering approximation and its strict analytical solution were derived.**

Keywords – Transport equation; light; scattering; Kubelka-Munk approach; multi-dimensional problem; analytical solution.

I. Introduction

Multi-dimensional (2D, 3D) radiative transfer remains a challenge in biomedical optics because the radiative transport equation (RTE) is difficult to solve analytically [1]. In spite of a number of existing approximations (diffusion, etc.), new approaches, especially extended Kubelka-Munk one (KM), are a subject of a study throughout all last decades [2-5]. However, the final, strict, and general analytical solution of the 3D or 2D problem has not been obtained. Most attempts completed by numerical methods or by a decomposition of fluxes in a power series. Probably, the root of the problem lies in the incorrect original RTE. Previously it was shown, that another form of RTE could exist for a slab geometry [4]. This study proposes a new approach for the 2D orthogonal-scattering model (OSM).

II. Materials and Methods

We used the multi-fluxes KM approach, similar to [5], with optical properties of the medium in a definition of Ref. [6]. We assumed a 2D OSM, where in both "x" and "y" direction two fluxes propagate only, forward f_+ and backward f_- (Fig. 1). In the medium, the light scattering takes place as backscattering or orthogonal-scattering ($f_x \leftrightarrow f_y$) events. It gives the following system of four differential equations:

$$\begin{cases} \frac{\partial f_{x+}(x,y)}{\partial x} = -\beta_1 f_{x+}(x,y) + \beta_2 f_{x-}(x,y) + \beta_3(f_{y-}(x,y) + f_{y+}(x,y)) \\ \frac{\partial f_{x-}(x,y)}{\partial x} = \beta_1 f_{x-}(x,y) - \beta_2 f_{x+}(x,y) - \beta_3(f_{y-}(x,y) + f_{y+}(x,y)) \\ \frac{\partial f_{y+}(x,y)}{\partial y} = -\beta_1 f_{y+}(x,y) + \beta_2 f_{y-}(x,y) + \beta_3(f_{x-}(x,y) + f_{x+}(x,y)) \\ \frac{\partial f_{y-}(x,y)}{\partial y} = \beta_1 f_{y-}(x,y) - \beta_2 f_{y+}(x,y) - \beta_3(f_{x-}(x,y) + f_{x+}(x,y)) \end{cases} \quad (1)$$

where β_1, β_2, β_3 are attenuation, backscattering, and lateral-scattering coefficients of the tested tissues respectively.

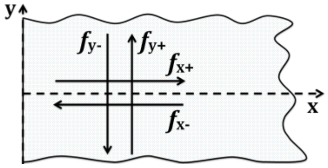

Fig.1. Fluxes inside the turbid tissue for OSM.

III. Results and Discussion

From the system (1), one can obtain the RTE for this task. Let $\alpha^2 = ({\beta_1}^2 - {\beta_2}^2)$ and $t = \sqrt{\alpha^2 + 2\beta_3(\beta_1 + \beta_2)}$. It yields:

$$\frac{\partial^4 f_i}{\partial x^2 \partial y^2} - \alpha^2 \left[\frac{\partial^2 f_i}{\partial y^2} + \frac{\partial^2 f_i}{\partial x^2}\right] + \left[\alpha^4 - 4{\beta_3}^2(\beta_1 + \beta_2)^2\right] f_i = 0 \quad (2)$$

with the analytical partial solution for each fluxes:

$$f_i(x,y) = (C_{i1}e^{tx} + C_{i2}e^{-tx}) \cdot (C_{i3}e^{ty} + C_{i4}e^{-ty}), \quad (3)$$

where C_{ik} – constants, which depend on boundary conditions.

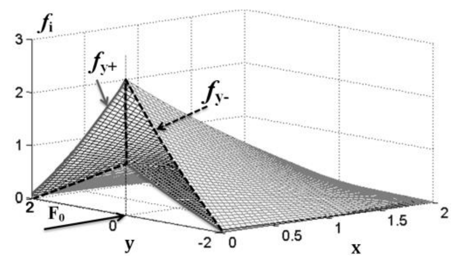

Fig. 2. Fluxes in the medium for the pencil-like central beam illumination with the flux power F_0.

IV. Conlusion

It was shown, that for OSM another RTE of the 4-th order exists contrary to the classic integro-differential RTE of the 1-th order. This new RTE has a strict analytical solution in a closed form for any 2D transfer problem in a limit of OSM.

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